

**Theorem 1 (Dominated convergence of Lebesgue)** Assume that  $g$  is an integrable function defined on the measurable set  $E$  and that  $(f_n)_{n \in \mathbb{N}}$  is a sequence of measurable functions so that  $|f_n| \leq g$ . If  $f$  is a function so that  $f_n \rightarrow f$  almost everywhere then

$$\lim_{n \rightarrow \infty} \int f_n = \int f.$$

Proof: The function  $g - f_n$  is non-negative and thus from Fatou lemma we have that  $\int (g - f) \leq \liminf \int (g - f_n)$ . Since  $|f| \leq g$  and  $|f_n| \leq g$  the functions  $f$  and  $f_n$  are integrable and we have

$$\int g - \int f \leq \int g - \limsup \int f_n,$$

so

$$\int f \geq \limsup \int f_n.$$

ABC

Figure 1: Caption in Sans fonts