

Package ‘hmmTensor’

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Type Package

Title Hidden Markov Model by Matrix and Tensor Decomposition

Version 0.1.0

Description Solves Hidden Markov Models (HMMs) via matrix and tensor decomposition. Converts observation sequences to co-occurrence matrices/tensors and applies Symmetric Non-negative Matrix Factorization (symNMF), Singular Value Decomposition (SVD), CANDECOMP/PARAFAC (CP) decomposition, or Tensor-Train (TT) decomposition to recover HMM parameters. Also provides standard HMM algorithms (Forward, Backward, Viterbi, Baum-Welch) for comparison. The spectral learning approach for HMMs is based on Hsu, Kakade, and Zhang (2012) <[doi:10.1016/j.jcss.2011.12.025](https://doi.org/10.1016/j.jcss.2011.12.025)>. The symNMF method is described in Kuang, Yun, and Park (2015) <[doi:10.1007/s10898-014-0247-2](https://doi.org/10.1007/s10898-014-0247-2)>. The Tensor-Train decomposition is described in Oseledets (2011) <[doi:10.1137/090752286](https://doi.org/10.1137/090752286)>.

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Imports rTensor, symTensor, methods, stats

Suggests testthat

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Backward	<i>Backward Algorithm for HMM</i>
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Description

Computes backward probabilities $\beta_t(i) = P(Y_{t+1}, \dots, Y_T | X_t = i)$ using the same scaling factors from the Forward algorithm.

Usage

```
Backward(Y, T_mat, O, scale)
```

Arguments

Y	Integer vector of observations (values in 1:N)
T_mat	Transition matrix (K x K), $T_mat[i, j] = P(X_t=j X_{t-1}=i)$
O	Emission matrix (K x N), $O[i, j] = P(Y_t=j X_t=i)$
scale	Scaling factors from Forward (length T_len)

Value

Matrix (K x T_len) of scaled backward probabilities

BaumWelch

*Baum-Welch Algorithm (EM) for HMM***Description**

Estimates HMM parameters (T, O, pi) from observation sequences using the Expectation-Maximization algorithm.

Usage

```
BaumWelch(
  Y,
  K,
  N,
  initT = NULL,
  initO = NULL,
  initPi = NULL,
  num.iter = 100L,
  thr = 1e-06,
  verbose = FALSE
)
```

Arguments

Y	Integer vector of observations (values in 1:N), or a list of integer vectors for multiple sequences.
K	Number of hidden states
N	Number of distinct observation symbols
initT	Initial transition matrix (K x K). If NULL, random initialization.
initO	Initial emission matrix (K x N). If NULL, random initialization.
initPi	Initial state distribution (length K). If NULL, uniform.
num.iter	Maximum EM iterations (default: 100)
thr	Convergence threshold on log-likelihood relative change (default: 1e-6)
verbose	Logical (default: FALSE)

Value

A list with components:

T_mat Estimated transition matrix (K x K)
O Estimated emission matrix (K x N)
pi0 Estimated initial distribution (length K)
loglik Vector of log-likelihoods per iteration
converged Logical
iter Number of iterations

 Forward

Forward Algorithm for HMM

Description

Computes forward probabilities $\alpha_t(i) = P(Y_1, \dots, Y_t, X_t = i)$ with log-scaling to avoid underflow.

Usage

```
Forward(Y, T_mat, O, pi0)
```

Arguments

Y	Integer vector of observations (values in 1:N)
T_mat	Transition matrix (K x K), $T_mat[i, j] = P(X_t=j X_{t-1}=i)$
O	Emission matrix (K x N), $O[i, j] = P(Y_t=j X_t=i)$
pi0	Initial state distribution (length K)

Value

A list with components:

alpha Matrix (K x T_len) of scaled forward probabilities

loglik Log-likelihood of the observation sequence

scale Vector of scaling factors (length T_len)

 HMM

HMM Parameter Estimation via Matrix/Tensor Decomposition

Description

Estimates HMM parameters by decomposing the observation co-occurrence matrix/tensor. Supports multiple decomposition solvers.

Usage

```
HMM(
  Y,
  K,
  N = NULL,
  solver = c("symNMF", "SVD", "CP", "TT"),
  Beta = 2,
  order = 2L,
```

```

    lag = 1L,
    smooth = 1e-10,
    num.iter = 100L,
    thr = 1e-10,
    verbose = FALSE
)

```

Arguments

Y	Integer vector of observations (values in 1:N), or a list of integer vectors for multiple sequences.
K	Number of hidden states
N	Number of distinct observation symbols. If NULL, inferred from data.
solver	Decomposition method: "symNMF" Symmetric NMF via <code>symTensor::symNMF</code> (default) "SVD" Truncated SVD of the co-occurrence matrix "CP" CP decomposition of the 3rd-order co-occurrence tensor via <code>rTensor::cp</code> "TT" Tensor-Train approximation (via Tucker + reshape)
Beta	Beta-divergence parameter for symNMF (default: 2). Ignored for other solvers.
order	Co-occurrence order for <code>Seq2Prob</code> : 2 (default) or 3. <code>order = 3</code> is required for <code>solver = "CP"</code> .
lag	Lag for pairwise co-occurrence (default: 1)
smooth	Laplace smoothing pseudo-count (default: 1e-10)
num.iter	Maximum iterations for iterative solvers (default: 100)
thr	Convergence threshold (default: 1e-10)
verbose	Logical (default: FALSE)

Details

The pipeline:

1. Convert observations to co-occurrence matrix $\Omega = P_{2,1}$ via [Seq2Prob](#)
2. Decompose Ω using the chosen solver
3. Recover HMM parameters (T, O, pi) from the decomposition

For NMF-based methods, $\Omega \approx M\Theta M^T$ where M relates to the emission matrix O and Θ to `diag(pi) %**% T`.

Value

A list with components:

- T_mat** Estimated transition matrix (K x K)
- O** Estimated emission matrix (K x N)
- pi0** Estimated initial distribution (length K)

Omega Co-occurrence matrix/tensor used
decomp Raw decomposition result
solver Solver used
RecError Reconstruction error (if available)

Examples

```
set.seed(42)
toy <- toyModel(type = "simple")
result <- HMM(toy$Y, K = toy$K, N = toy$N, solver = "symNMF")
result$T_mat
```

Seq2Prob

Convert Observation Sequences to Co-occurrence Matrix/Tensor

Description

Constructs empirical co-occurrence statistics from observation sequences. These form the basis for matrix/tensor decomposition approaches to HMM.

Usage

```
Seq2Prob(Y, N = NULL, order = 2L, lag = 1L, smooth = 0)
```

Arguments

Y	Integer vector of observations (values in 1:N), or a list of integer vectors for multiple sequences.
N	Number of distinct observation symbols. If NULL, inferred from data.
order	Co-occurrence order: 2 (pairwise, default) or 3 (triple). Order 2 gives an N x N matrix $P_{2,1}(i, j) = P(Y_2 = i, Y_1 = j)$. Order 3 gives an N x N x N tensor $P_3(i, j, k) = P(Y_3 = i, Y_2 = j, Y_1 = k)$.
lag	Lag for pairwise co-occurrence (default: 1). $\Omega^{(\tau)}(i, j) = P(Y_t = i, Y_{t+\tau} = j)$. Only used when order = 2.
smooth	Laplace smoothing pseudo-count (default: 0). Adds smooth to all counts before normalization.

Value

For order = 2: an N x N matrix (normalized). For order = 3: an rTensor Tensor object of dimension N x N x N.

Examples

```
Y <- c(1, 2, 3, 1, 2, 3, 1, 2, 3)
P2 <- Seq2Prob(Y, N = 3, order = 2)
P3 <- Seq2Prob(Y, N = 3, order = 3)
```

toyModel	<i>Generate Toy HMM Data</i>
----------	------------------------------

Description

Creates synthetic HMM data with visually clear structure for demonstrations and testing.

Usage

```
toyModel(type = c("simple", "weather", "leftright"), T_len = 500L, seed = NULL)
```

Arguments

type	Type of toy model: "simple" 2 states, 3 observations. States alternate with high probability. Clear emission separation. "weather" 3 states (Sunny/Cloudy/Rainy), 4 observations (Walk/Shop/Clean/Stay). Classic weather HMM. "leftright" 3 states with left-to-right transitions only. Models sequential processes.
T_len	Length of observation sequence (default: 500)
seed	Random seed (default: NULL)

Value

A list with components:

Y Integer vector of observations
X Integer vector of true hidden states
T_mat True transition matrix
O True emission matrix
pi0 True initial distribution
K Number of hidden states
N Number of observation symbols
type Model type

Examples

```
toy <- toyModel("simple", T_len = 200, seed = 42)  
table(toy$X)  
table(toy$Y)
```

Viterbi

Viterbi Algorithm for HMM

Description

Finds the most likely state sequence given the observations using the Viterbi algorithm (log-space).

Usage

```
Viterbi(Y, T_mat, O, pi0)
```

Arguments

Y	Integer vector of observations (values in 1:N)
T_mat	Transition matrix (K x K), $T_mat[i, j] = P(X_t=j X_{t-1}=i)$
O	Emission matrix (K x N), $O[i, j] = P(Y_t=j X_t=i)$
pi0	Initial state distribution (length K)

Value

A list with components:

path Integer vector of most likely states (length T_len)

loglik Log-likelihood of the best path

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