Equations for the rstpm2 package

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Contents

1 Generalised survival model

```
logexpand:all;
derivsubst:true;
load(pdiff)$
assume(t>0)$
S : S0(t)*G(eta(t,z,theta));
H : -log(S);
h : diff(H,t);
print("H:");
tex(H);
print("h:");
tex(h);

print("diff(H,theta):");
```

```
tex(diff(H,theta));

print("diff(log(h),theta):");
tex(diff(log(h),theta));

print("ll:");
ll : delta*log(h) - H -  diff(eta(t,z,beta),t)^2;
tex(ll);
print("diff(ll, beta)");
tex(diff(ll, beta));
```

H:

$$-\log G\left(\eta\left(t,z,\vartheta\right)\right)-\log S_0\left(t\right)$$

h:

$$-\frac{\eta_{(1,0,0)}(t,z,\vartheta)\,G_{(1)}(\eta\left(t,z,\vartheta\right))}{G\left(\eta\left(t,z,\vartheta\right)\right)}-\frac{S_{0(1)}(t)}{S_0\left(t\right)}$$

diff(H,theta):

$$-\frac{\eta_{(0,0,1)}(t,z,\vartheta)\,G_{(1)}(\eta\left(t,z,\vartheta\right))}{G\left(\eta\left(t,z,\vartheta\right)\right)}$$

diff(log(h),theta):

$$\frac{-\frac{\eta_{(0,0,1)}(t,z,\vartheta)\,\eta_{(1,0,0)}(t,z,\vartheta)\,G_{(2)}(\eta(t,z,\vartheta))}{G(\eta(t,z,\vartheta))}+\frac{\eta_{(0,0,1)}(t,z,\vartheta)\,\eta_{(1,0,0)}(t,z,\vartheta)\,G_{(1)}(\eta(t,z,\vartheta))^2}{G(\eta(t,z,\vartheta))^2}-\frac{\eta_{(1,0,1)}(t,z,\vartheta)\,G_{(1)}(\eta(t,z,\vartheta))}{G(\eta(t,z,\vartheta))}}{-\frac{\eta_{(1,0,0)}(t,z,\vartheta)\,G_{(1)}(\eta(t,z,\vartheta))}{G(\eta(t,z,\vartheta))}-\frac{S_{0(1)}(t)}{S_0(t)}}$$

ll:

$$\delta\,\log\left(-\frac{\eta_{(1,0,0)}(t,z,\vartheta)\,G_{(1)}(\eta\left(t,z,\vartheta\right))}{G\left(\eta\left(t,z,\vartheta\right)\right)}-\frac{S_{0(1)}(t)}{S_0\left(t\right)}\right)+\log G\left(\eta\left(t,z,\vartheta\right)\right)+\log S_0\left(t\right)-\eta_{(1,0,0)}(t,z,\beta)^2$$

diff(ll, beta)

$$-2\,\eta_{(1,0,0)}(t,z,\beta)\,\eta_{(1,0,1)}(t,z,\beta)$$

2 Mixture cure models

```
logexpand:all;
derivsubst:true;
load(pdiff)$
S : S0(t)*(pi(theta)+(1-pi(theta))*exp(-Hu(t,theta)));
H : -log(S);
h : diff(H,t);
h : subst(hu(t,theta), diff(Hu(t,theta),t), h);
```

```
print("H:");
tex(H);
print("h:");
tex(h);

print("diff(H,theta):");
tex(diff(H,theta));

print("diff(log(h),theta):");
tex(diff(log(h),theta));
diff(log(h),theta) - diff(H,theta);
```

H:

$$-\log \left(\pi\left(\vartheta\right)+e^{-Hu\left(t,\vartheta\right)}\,\left(1-\pi\left(\vartheta\right)\right)\right)-\log S_{0}\left(t\right)$$

h:

$$\frac{e^{-Hu\left(t,\vartheta\right)}\,hu\left(t,\vartheta\right)\,\left(1-\pi\left(\vartheta\right)\right)}{\pi\left(\vartheta\right)+e^{-Hu\left(t,\vartheta\right)}\,\left(1-\pi\left(\vartheta\right)\right)}-\frac{S_{0(1)}(t)}{S_{0}\left(t\right)}$$

diff(H,theta):

$$-\frac{-e^{-Hu\left(t,\vartheta\right)}\,\pi_{(1)}(\vartheta)+\pi_{(1)}(\vartheta)-e^{-Hu\left(t,\vartheta\right)}\,Hu_{(0,1)}(t,\vartheta)\,\left(1-\pi\left(\vartheta\right)\right)}{\pi\left(\vartheta\right)+e^{-Hu\left(t,\vartheta\right)}\,\left(1-\pi\left(\vartheta\right)\right)}$$

diff(log(h),theta):

$$\frac{-\frac{e^{-Hu(t,\vartheta)}\,hu(t,\vartheta)\,(1-\pi(\vartheta))\,\left(-e^{-Hu(t,\vartheta)}\,\pi_{(1)}(\vartheta)+\pi_{(1)}(\vartheta)-e^{-Hu(t,\vartheta)}\,Hu_{(0,1)}(t,\vartheta)\,(1-\pi(\vartheta))\right)}{\left(\pi(\vartheta)+e^{-Hu(t,\vartheta)}\,(1-\pi(\vartheta))\right)^2}-\frac{e^{-Hu(t,\vartheta)}\,hu(t,\vartheta)\,\pi_{(1)}(\vartheta)}{\pi(\vartheta)+e^{-Hu(t,\vartheta)}\,(1-\pi(\vartheta))}+\frac{e^{-Hu(t,\vartheta)}\,hu_{(0,1)}(t,\vartheta)\,(1-\pi(\vartheta))}{\pi(\vartheta)+e^{-Hu(t,\vartheta)}\,(1-\pi(\vartheta))}-\frac{e^{-Hu(t,\vartheta)}\,Hu_{(0,1)}(t,\vartheta)\,hu(t,\vartheta)\,(1-\pi(\vartheta))}{\pi(\vartheta)+e^{-Hu(t,\vartheta)}\,(1-\pi(\vartheta))}}{\frac{e^{-Hu(t,\vartheta)}\,hu(t,\vartheta)\,(1-\pi(\vartheta))}{\pi(\vartheta)+e^{-Hu(t,\vartheta)}\,(1-\pi(\vartheta))}-\frac{S_{0(1)}(t)}{S_0(t)}}$$

3 Integral equation for AFT models

```
logexpand:all;
derivsubst:true;
load(pdiff)$
assume(t>0)$
S : exp(-exp(B(log(integrate(exp(-x(v)*beta),v,0,t)),gamma)));
H : -log(S);
h : diff(H,t);
print("H:");
tex(H);
print("h:");
tex(h);
```

```
print("log(h):");
tex(log(h));

print("diff(H,beta):");
tex(diff(H,beta));
print("diff(H,gamma):");
tex(diff(H,gamma));

print("diff(log(h),beta):");
tex(diff(log(h),beta));
print("diff(log(h),gamma):");
tex(diff(log(h),gamma));

print("ll:");
ll : delta*log(h) - H - (subst(u=log(integrate(exp(-x(v)*beta),v,0,t)), diff(B(u,gamma),u)))^2;
tex(ll);
print("diff(ll, beta)");
tex(diff(ll, beta));
print("diff(ll, gamma)");
tex(diff(ll, gamma));
```

H:

$$e^{B\left(\log \int_{0}^{t}{e^{-\beta\,x\left(v\right)}\;dv} , \gamma\right)}$$

h:

$$\frac{e^{B\left(\log \int_{0}^{t}{e^{-\beta\,x\left(v\right)}\;dv} , \gamma\right)-\beta\,x\left(t\right)}\,B_{(1,0)}(\log \int_{0}^{t}{e^{-\beta\,x\left(v\right)}\;dv} , \gamma)}{\int_{0}^{t}{e^{-\beta\,x\left(v\right)}\;dv}}$$

log(h):

$$\log B_{(1,0)}(\log \int_{0}^{t}{e^{-\beta\,x\left(v\right)}\;dv} , \gamma)+B\left(\log \int_{0}^{t}{e^{-\beta\,x\left(v\right)}\;dv} , \gamma\right)-\log \int_{0}^{t}{e^{-\beta\,x\left(v\right)}\;dv}-\beta\,x\left(t\right)$$

diff(H,beta):

$$-\frac{\left(\int_{0}^{t}{e^{-\beta\,x\left(v\right)}\,x\left(v\right)\;dv}\right)\,e^{B\left(\log \int_{0}^{t}{e^{-\beta\,x\left(v\right)}\;dv} , \gamma\right)}\,B_{(1,0)}(\log \int_{0}^{t}{e^{-\beta\,x\left(v\right)}\;dv} , \gamma)}{\int_{0}^{t}{e^{-\beta\,x\left(v\right)}\;dv}}$$

diff(H,gamma):

$$e^{B\left(\log \int_{0}^{t}{e^{-\beta\,x\left(v\right)}\;dv} , \gamma\right)}\,B_{(0,1)}(\log \int_{0}^{t}{e^{-\beta\,x\left(v\right)}\;dv} , \gamma)$$

diff(log(h),beta):

$$-\frac{\left(\int_{0}^{t}{e^{-\beta\,x\left(v\right)}\,x\left(v\right)\;dv}\right)\,B_{(2,0)}(\log \int_{0}^{t}{e^{-\beta\,x\left(v\right)}\;dv} , \gamma)}{\left(\int_{0}^{t}{e^{-\beta\,x\left(v\right)}\;dv}\right)\,B_{(1,0)}(\log \int_{0}^{t}{e^{-\beta\,x\left(v\right)}\;dv} , \gamma)}-\frac{\left(\int_{0}^{t}{e^{-\beta\,x\left(v\right)}\,x\left(v\right)\;dv}\right)\,B_{(1,0)}(\log \int_{0}^{t}{e^{-\beta\,x\left(v\right)}\;dv} , \gamma)}{\int_{0}^{t}{e^{-\beta\,x\left(v\right)}\;dv}}+\frac{\int_{0}^{t}{e^{-\beta\,x\left(v\right)}\,x\left(v\right)\;dv}}{\int_{0}^{t}{e^{-\beta\,x\left(v\right)}\;dv}}-x\left(t\right)$$

diff(log(h),gamma):

$$\frac{B_{(1,1)}(\log \int_0^t e^{-\beta\, x(v)}\; dv, \gamma)}{B_{(1,0)}(\log \int_0^t e^{-\beta\, x(v)}\; dv, \gamma)} + B_{(0,1)}(\log \int_0^t e^{-\beta\, x(v)}\; dv, \gamma)$$

ll:

$$\delta\left(\log B_{(1,0)}(\log \int_0^t e^{-\beta\, x(v)}\; dv, \gamma) + B\left(\log \int_0^t e^{-\beta\, x(v)}\; dv, \gamma\right) - \log \int_0^t e^{-\beta\, x(v)}\; dv - \beta\, x\left(t\right)\right) - B_{(1,0)}(\log \int_0^t e^{-\beta\, x(v)}\; dv, \gamma)^2 - e^{B\left(\log \int_0^t e^{-\beta\, x(v)}\; dv, \gamma\right)}$$

diff(ll, beta)

$$\delta\left(-\frac{\left(\int_0^t e^{-\beta\, x(v)}\, x\left(v\right)\; dv\right)\, B_{(2,0)}(\log \int_0^t e^{-\beta\, x(v)}\; dv, \gamma)}{\left(\int_0^t e^{-\beta\, x(v)}\; dv\right)\, B_{(1,0)}(\log \int_0^t e^{-\beta\, x(v)}\; dv, \gamma)} - \frac{\left(\int_0^t e^{-\beta\, x(v)}\, x\left(v\right)\; dv\right)\, B_{(1,0)}(\log \int_0^t e^{-\beta\, x(v)}\; dv, \gamma)}{\int_0^t e^{-\beta\, x(v)}\; dv} + \frac{\int_0^t e^{-\beta\, x(v)}\, x\left(v\right)\; dv}{\int_0^t e^{-\beta\, x(v)}\; dv} - x\left(t\right)\right) + \frac{2\, \left(\int_0^t e^{-\beta\, x(v)}\, x\left(v\right)\; dv\right)}{}$$

diff(ll, gamma)

$$\delta\left(\frac{B_{(1,1)}(\log \int_0^t e^{-\beta\, x(v)}\; dv, \gamma)}{B_{(1,0)}(\log \int_0^t e^{-\beta\, x(v)}\; dv, \gamma)} + B_{(0,1)}(\log \int_0^t e^{-\beta\, x(v)}\; dv, \gamma)\right) - 2\, B_{(1,0)}(\log \int_0^t e^{-\beta\, x(v)}\; dv, \gamma)\, B_{(1,1)}(\log \int_0^t e^{-\beta\, x(v)}\; dv, \gamma) - e^{B\left(\log \int_0^t e^{-\beta\, x(v)}\; dv, \gamma\right)}\, B_{(0,1)}(\log \int_0^t e^{-\beta\, x(v)}$$

4 Integral equation for AFT models with functional constraints on gamma

```
logexpand:all;
derivsubst:true;
load(pdiff)$
assume(t>0)$
gamma : gamma0 + exp(alpha);
S : exp(-exp(B(log(integrate(exp(-x(v)*beta),v,0,t)),gamma)));
H : -log(S);
h : diff(H,t);
print("H:");
tex(H);
print("h:");
tex(h);

print("diff(H,beta):");
tex(diff(H,beta));
print("diff(H,alpha):");
tex(diff(H,alpha));
print("diff(H,gamma0):");
tex(diff(H,gamma0));
```

```
print("diff(log(h),beta):");
tex(diff(log(h),beta));
print("diff(log(h),alpha):");
tex(diff(log(h),alpha));
print("diff(log(h),gamma0):");
tex(diff(log(h),gamma0));
```

H:

$$e^{B\left(\log \int_0^t e^{-\beta\,x(v)}\;dv,\gamma_0+e^\alpha\right)}$$

h:

$$\frac{e^{B\left(\log \int_0^t e^{-\beta\,x(v)}\;dv,\gamma_0+e^\alpha\right)-\beta\,x(t)}\,B_{(1,0)}(\log \int_0^t e^{-\beta\,x(v)}\;dv,\gamma_0+e^\alpha)}{\int_0^t e^{-\beta\,x(v)}\;dv}$$

diff(H,beta):

$$-\frac{\left(\int_0^t e^{-\beta\,x(v)}\,x\left(v\right)\;dv\right)\,e^{B\left(\log \int_0^t e^{-\beta\,x(v)}\;dv,\gamma_0+e^\alpha\right)}\,B_{(1,0)}(\log \int_0^t e^{-\beta\,x(v)}\;dv,\gamma_0+e^\alpha)}{\int_0^t e^{-\beta\,x(v)}\;dv}$$

diff(H,alpha):

$$e^{B\left(\log \int_0^t e^{-\beta\,x(v)}\;dv,\gamma_0+e^\alpha\right)+\alpha}\,B_{(0,1)}(\log \int_0^t e^{-\beta\,x(v)}\;dv,\gamma_0+e^\alpha)$$

diff(H,gamma0):

$$e^{B\left(\log \int_0^t e^{-\beta\,x(v)}\;dv,\gamma_0+e^\alpha\right)}\,B_{(0,1)}(\log \int_0^t e^{-\beta\,x(v)}\;dv,\gamma_0+e^\alpha)$$

diff(log(h),beta):

$$-\frac{\left(\int_0^t e^{-\beta\,x(v)}\,x\left(v\right)\;dv\right)\,B_{(2,0)}(\log \int_0^t e^{-\beta\,x(v)}\;dv,\gamma_0+e^\alpha)}{\left(\int_0^t e^{-\beta\,x(v)}\;dv\right)\,B_{(1,0)}(\log \int_0^t e^{-\beta\,x(v)}\;dv,\gamma_0+e^\alpha)}-\frac{\left(\int_0^t e^{-\beta\,x(v)}\,x\left(v\right)\;dv\right)\,B_{(1,0)}(\log \int_0^t e^{-\beta\,x(v)}\;dv,\gamma_0+e^\alpha)}{\int_0^t e^{-\beta\,x(v)}\;dv}+\frac{\int_0^t e^{-\beta\,x(v)}\,x\left(v\right)\;dv}{\int_0^t e^{-\beta\,x(v)}\;dv}-x\left(t\right)$$

diff(log(h),alpha):

$$\frac{e^\alpha\,B_{(1,1)}(\log \int_0^t e^{-\beta\,x(v)}\;dv,\gamma_0+e^\alpha)}{B_{(1,0)}(\log \int_0^t e^{-\beta\,x(v)}\;dv,\gamma_0+e^\alpha)}+e^\alpha\,B_{(0,1)}(\log \int_0^t e^{-\beta\,x(v)}\;dv,\gamma_0+e^\alpha)$$

diff(log(h),gamma0):

$$\frac{B_{(1,1)}(\log \int_0^t e^{-\beta\,x(v)}\;dv,\gamma_0+e^\alpha)}{B_{(1,0)}(\log \int_0^t e^{-\beta\,x(v)}\;dv,\gamma_0+e^\alpha)}+B_{(0,1)}(\log \int_0^t e^{-\beta\,x(v)}\;dv,\gamma_0+e^\alpha)$$

5 Non-integral equation for AFT models

```
logexpand:all;
derivsubst:true;
load(pdiff)$
assume(t>0)$
S : exp(-exp(B(log(t*exp(-eta(X,log(t),beta))),gamma)));
H : -log(S);
h : diff(H,t);
print("H:");
tex(H);
print("h:");
tex(h);
print("log(h):");
tex(log(h));

print("diff(H,beta):");
tex(diff(H,beta));
print("diff(H,gamma):");
tex(diff(H,gamma));

print("diff(log(h),beta):");
tex(diff(log(h),beta));
print("diff(log(h),gamma):");
tex(diff(log(h),gamma));

print("ll:");
ll : delta*log(h) - H - (1-subst(u=log(t), diff(eta(X,u,beta),u)))^2 -
subst(u=log(t)-eta(X,log(t),beta), diff(B(u,gamma),u))^2;
tex(ll);
print("diff(ll, beta)");
tex(diff(ll, beta));
print("diff(ll, gamma)");
tex(diff(ll, gamma));
```

H:

$$e^{B(\log t-\eta(X,\log t,\beta),\gamma)}$$

h:

$$\left(\frac{1}{t}-\frac{\eta_{(0,1,0)}(X,\log t,\beta)}{t}\right)\,e^{B(\log t-\eta(X,\log t,\beta),\gamma)}\,B_{(1,0)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)$$

log(h):

$$\log B_{(1,0)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)+B\left(\log t-\eta\left(X,\log t,\beta\right),\gamma\right)+\log\left(\frac{1}{t}-\frac{\eta_{(0,1,0)}(X,\log t,\beta)}{t}\right)$$

diff(H,beta):

$$-\eta_{(0,0,1)}(X,\log t,\beta)\,e^{B(\log t-\eta(X,\log t,\beta),\gamma)}\,B_{(1,0)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)$$

diff(H,gamma):

$$e^{B(\log t-\eta(X,\log t,\beta),\gamma)}\,B_{(0,1)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)$$

diff(log(h),beta):

$$-\frac{\eta_{(0,0,1)}(X,\log t,\beta)\,B_{(2,0)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)}{B_{(1,0)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)}-\eta_{(0,0,1)}(X,\log t,\beta)\,B_{(1,0)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)-\frac{\eta_{(0,1,1)}(X,\log t,\beta)}{\left(\frac{1}{t}-\frac{\eta_{(0,1,0)}(X,\log t,\beta)}{t}\right)\,t}$$

diff(log(h),gamma):

$$\frac{B_{(1,1)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)}{B_{(1,0)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)}+B_{(0,1)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)$$

ll:

$$\delta\,\left(\log B_{(1,0)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)+B\left(\log t-\eta\left(X,\log t,\beta\right),\gamma\right)+\log\left(\frac{1}{t}-\frac{\eta_{(0,1,0)}(X,\log t,\beta)}{t}\right)\right)-B_{(1,0)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)^2-e^{B(\log t-\eta(X,\log t,\beta),\gamma)}-(1-\eta$$

diff(ll, beta)

$$\delta\,\left(-\frac{\eta_{(0,0,1)}(X,\log t,\beta)\,B_{(2,0)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)}{B_{(1,0)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)}-\eta_{(0,0,1)}(X,\log t,\beta)\,B_{(1,0)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)-\frac{\eta_{(0,1,1)}(X,\log t,\beta)}{\left(\frac{1}{t}-\frac{\eta_{(0,1,0)}(X,\log t,\beta)}{t}\right)\,t}\right)+2\,\eta_{(0,0,1)}(X,\log t,\beta)\,B_{(1,0)}($$

diff(ll, gamma)

$$\delta\,\left(\frac{B_{(1,1)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)}{B_{(1,0)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)}+B_{(0,1)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)\right)-2\,B_{(1,0)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)\,B_{(1,1)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)-e^{B(\log t-\eta(X,\log t,\beta),\gamma)}\,B_{(0,1)}(\log$$

6 Non-integral equation for AFT models without time-varying effect

```
logexpand:all;
derivsubst:true;
load(pdiff)$
assume(t>0)$
S : exp(-exp(B(log(t*exp(-eta(X,beta))),gamma)));
H : -log(S);
h : diff(H,t);
```

```
print("H:");
tex(H);
print("h:");
tex(h);
print("log(h):");
tex(log(h));

print("diff(H,beta):");
tex(diff(H,beta));
print("diff(H,gamma):");
tex(diff(H,gamma));

print("diff(log(h),beta):");
tex(diff(log(h),beta));
print("diff(log(h),gamma):");
tex(diff(log(h),gamma));

print("ll:");
ll : delta*log(h) - H - (1-subst(u=log(t), diff(eta(X,u,beta),u)))^2 -
subst(u=log(t)-eta(X,log(t),beta), diff(B(u,gamma),u))^2;
tex(ll);
print("diff(ll, beta)");
tex(diff(ll, beta));
print("diff(ll, gamma)");
tex(diff(ll, gamma));
```

H:

$$e^{B(\log t-\eta(X,\beta),\gamma)}$$

h:

$$\frac{e^{B(\log t-\eta(X,\beta),\gamma)}\,B_{(1,0)}(\log t-\eta\left(X,\beta\right),\gamma)}{t}$$

log(h):

$$\log B_{(1,0)}(\log t-\eta\left(X,\beta\right),\gamma)+B\left(\log t-\eta\left(X,\beta\right),\gamma\right)-\log t$$

diff(H,beta):

$$-\eta_{(0,1)}(X,\beta)\,e^{B(\log t-\eta(X,\beta),\gamma)}\,B_{(1,0)}(\log t-\eta\left(X,\beta\right),\gamma)$$

diff(H,gamma):

$$e^{B(\log t-\eta(X,\beta),\gamma)}\,B_{(0,1)}(\log t-\eta\left(X,\beta\right),\gamma)$$

diff(log(h),beta):

$$-\frac{\eta_{(0,1)}(X,\beta)\,B_{(2,0)}(\log t-\eta\left(X,\beta\right),\gamma)}{B_{(1,0)}(\log t-\eta\left(X,\beta\right),\gamma)}-\eta_{(0,1)}(X,\beta)\,B_{(1,0)}(\log t-\eta\left(X,\beta\right),\gamma)$$

diff(log(h),gamma):

$$\frac{B_{(1,1)}(\log t-\eta\left(X,\beta\right),\gamma)}{B_{(1,0)}(\log t-\eta\left(X,\beta\right),\gamma)}+B_{(0,1)}(\log t-\eta\left(X,\beta\right),\gamma)$$

ll:

$$\delta\,\left(\log B_{(1,0)}(\log t-\eta\left(X,\beta\right),\gamma)+B\left(\log t-\eta\left(X,\beta\right),\gamma\right)-\log t\right)-B_{(1,0)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)^2-e^{B\left(\log t-\eta\left(X,\beta\right),\gamma\right)}-\left(1-\eta_{(0,1,0)}(X,\log t,\beta)\right)^2$$

diff(ll, beta)

$$2\,\eta_{(0,0,1)}(X,\log t,\beta)\,B_{(1,0)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)\,B_{(2,0)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)+\delta\,\left(-\frac{\eta_{(0,1)}(X,\beta)\,B_{(2,0)}(\log t-\eta\left(X,\beta\right),\gamma)}{B_{(1,0)}(\log t-\eta\left(X,\beta\right),\gamma)}-\eta_{(0,1)}(X,\beta)\,B_{(1,0)}(\log t-\eta\left(X,\beta\right),\gamma)$$

diff(ll, gamma)

$$-2\,B_{(1,0)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)\,B_{(1,1)}(\log t-\eta\left(X,\log t,\beta\right),\gamma)+\delta\,\left(\frac{B_{(1,1)}(\log t-\eta\left(X,\beta\right),\gamma)}{B_{(1,0)}(\log t-\eta\left(X,\beta\right),\gamma)}+B_{(0,1)}(\log t-\eta\left(X,\beta\right),\gamma)\right)-e^{B\left(\log t-\eta\left(X,\beta\right),\gamma\right)}\,B_{(0,1)}(\log t-\eta\left(X,\beta\right),$$

7 Integral equations for AFT models with $H = B(\ldots)$

```
logexpand:all;
derivsubst:true;
load(pdiff)$
assume(t>0)$
S : exp(-B(integrate(exp(-x(v)*beta),v,0,t),gamma));
H : -log(S);
h : diff(H,t);
print("H:");
tex(H);
print("h:");
tex(h);

print("diff(H,beta):");
tex(diff(H,beta));
print("diff(H,gamma):");
tex(diff(H,gamma));

print("diff(log(h),beta):");
tex(diff(log(h),beta));
print("diff(log(h),gamma):");
tex(diff(log(h),gamma));
```

H:

$$B\left(\int_0^t e^{-\beta\,x(v)}\;dv,\gamma\right)$$

h:

$$e^{-\beta\,x(t)}\,B_{(1,0)}(\int_0^t e^{-\beta\,x(v)}\;dv,\gamma)$$

diff(H,beta):

$$-\left(\int_0^t e^{-\beta\,x(v)}\,x\left(v\right)\;dv\right)\,B_{(1,0)}(\int_0^t e^{-\beta\,x(v)}\;dv,\gamma)$$

diff(H,gamma):

$$B_{(0,1)}(\int_0^t e^{-\beta\,x(v)}\;dv,\gamma)$$

diff(log(h),beta):

$$-\frac{\left(\int_0^t e^{-\beta\,x(v)}\,x\left(v\right)\;dv\right)\,B_{(2,0)}(\int_0^t e^{-\beta\,x(v)}\;dv,\gamma)}{B_{(1,0)}(\int_0^t e^{-\beta\,x(v)}\;dv,\gamma)}-x\left(t\right)$$

diff(log(h),gamma):

$$\frac{B_{(1,1)}(\int_0^t e^{-\beta\,x(v)}\;dv,\gamma)}{B_{(1,0)}(\int_0^t e^{-\beta\,x(v)}\;dv,\gamma)}$$